What does the Existence & Uniqueness Theorem tell you about the IVP $(\sin x)y' - y^{\frac{5}{2}} = 0, \quad y(\frac{\pi}{4}) = 0$? **SCORE: _____ / 3 PTS**

Justify your answer properly, but briefly.

($\frac{1}{2}$) $y' = \frac{y^{\frac{5}{2}}}{\sin x} = f$, so $f_y = \frac{\frac{5}{2}y^{\frac{3}{2}}}{\sin x}$ (1) WHICH IS NOT DEFINED/CONTINUOUS AROUND $(\frac{\pi}{4}, 0)$ WHERE $y < 0$ ($\frac{1}{2}$)

SO E+U TELLS US NOTHING (1)

Consider the DE $x^2 \frac{d^2 y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^2$. SCORE: _____ / 6 PTS

[a] Is $y = Ax^3 + x^2 + Bx$ a family of solutions of the DE ?

$y' = 3Ax^2 + 2x + B$

$y'' = 6Ax + 2$

$x^2 y'' - 2xy' + 2y = x^2(6Ax+2) - 2x(3Ax^2+2x+B) + 2(Ax^3+x^2+Bx)$ (1) (1) (1)

$= 2Ax^3$ $(1\frac{1}{2})$

NO $(1\frac{1}{2})$

[b] If the answer to [a] is "YES", solve the IVP consisting of the DE and the initial conditions $y(1) = -1, \quad y'(1) = 3$.
If the answer to [a] is "NO", skip this part.

MULTIPLE CHOICE. Consider the DEs **SCORE: _____ / 3 PTS**

[1] $(5r+1)\dfrac{dr}{d\theta}=2\theta+1$ [2] $x''y-y^2=2x$ [3] $(5w+1)du+(\ln w-3u)dw=0$

(where w is the independent variable)

Which of the DE above are linear ? Circle the correct answer below.

(a) none are linear (b) only [1] is linear (c) only [2] is linear (d) only [3] is linear

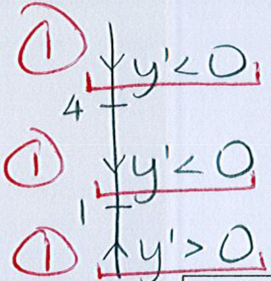
(e) only [1] & [2] are linear (f) only [1] & [3] are linear (g) only [2] & [3] are linear (h) all are linear

3

Consider the autonomous DE $y' = (y-4)^2(1-y)$. **SCORE: _____ / 7 PTS**

[a] Find all equilibrium solutions of the DE and classify each as stable, unstable or semi-stable.

$(y-4)^2(1-y)=0 \longrightarrow y=1,4$



$y=4$ SEMI-STABLE (1)

$y=1$ STABLE (1)

[b] If $y=f(x)$ is a solution of the DE such that $f(2)=5$, what is $\lim_{x\to\infty} f(x)$? HINT: Sketch a possible graph of $y=f(x)$.

4 (1)

[c] If $y=g(x)$ is a solution of the DE such that $g(5)=-3$, what is $\lim_{x\to\infty} g(x)$?

1 (1)

FILL IN THE BLANKS. SCORE: _____ / 3 PTS

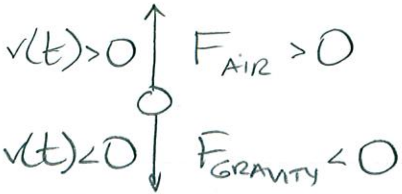
[a] The order of the DE $y^{10} - y^{(7)}y^4 = (x^5 + y''')^6$ is 7. (1)

[b] If $y = \sqrt{x+9}$ is a solution of the DE $y'' = f(x, y, y')$, the largest possible interval of definition is $(-9, \infty)$. (2)

$y' = \frac{1}{2}(x+9)^{-\frac{1}{2}}$ $y'' = -\frac{1}{4}(x+9)^{-\frac{3}{2}}$

NOTE: −9 IS NOT IN THE INTERVAL

Write a differential equation for the velocity $v(t)$ of a falling object if the air resistance is proportional to the square of the velocity. Assume that $v(t) > 0$ corresponds to the object moving upward, $v(t) < 0$ corresponds to the object moving downward. (NOTE: This is NOT the same problem as in the homework.) **SCORE: _____ / 3 PTS**



$$m\frac{dv}{dt} = -mg + kv^2$$

(1) (1) (1)

MUST HAVE CORRECT SIGN

Consider the IVP $y' = 5x - 10y, \quad y(1) = -2$. **SCORE: _____ / 5 PTS**

Use Euler's method with $h = 0.2$ to estimate $y(1.4)$.

$y(1.2) \approx y(1) + y'(1)(0.2) = -2 + (5(1) - 10(-2))(0.2)$

$= -2 + 25(0.2) = 3$ (2) (1)

$y(1.4) \approx y(1.2) + y'(1.2)(0.2) \approx 3 + (-24)(0.2) = -1.8$ (1) (1)